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STATISTICAL MECHANICS OF GAS SUSPENSIONS. A QUASI-ISOTROPIC MODEL

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A statistical theory of streams of systems of the gas + suspended particles type is proposed. The theory is based on the assumption that the particle concentration fluctuations are isotropic. The structure of the equilibrium states is considered in a gradientless approximation; the mean-square values of the pulsations of the dynamic quantities and the transfer coefficients are estimated; the size of the local inhomogeneities is determined. Equations for the energy of the pulsations of the dispersed phase in various directions are obtained; an energy equation complementing the system of dynamic equations given in [1] is derived.

The statistical characteristics of random pulsating motions of the phases in a gas suspension stream can be found by solving the system of integrodifferential spectral equations obtained in [1]. This system is quite complicated. It is therefore expedient to make use of some simplifying hypotheses; this makes it possible to reduce the stochastic equations of [1] to the equations of [2].

1. The pulsation equations. Let us consider the motion of a monodisperse gas suspension under the assumption that the time and space scales of variation of the mean parameters describing the flow (of the "dynamic quantities") are large as compared with the scales of the local pulsations. This enables us, among other things, to carry out our computations in a coordinate system in which the velocity of the dispersed phase in the volume element under consideration is equal to zero.

Let us make use of the most "fine-grained" description of the pulsations of dynamic quantities permitted by the notion of phases as interacting interpenetrating continuous media, i.e. let us choose as our characteristic physical volume (the "averaging scale" in the terminology of [1] the specific volume $\sigma = l^3$ of a single suspended particle [1,2]. In accordance with the above assumption we neglect the derivatives of the dynamic quantities with respect to time and the coordinates as compared with the corresponding derivatives of the fluctuations of these quantities. This approximation is analogous in meaning to the familiar hydrodynamic approximation of kinetic theory [2]. We then have the following equations for the pulsations of the mean parameters (the notation is that of [1]

$$\left(\frac{\partial}{\partial t} + \mathbf{u}\frac{\partial}{\partial \mathbf{r}}\right)\rho_l' - (1-\rho)\frac{\partial \mathbf{v}_l'}{\partial \mathbf{r}} = Q_l^{(g)}, \qquad \frac{\partial \rho_l'}{\partial t} + \rho\frac{\partial \mathbf{w}_l}{\partial \mathbf{r}} = Q_l^{(p)}$$

$$-(1-\rho)\frac{\partial \pi_l'}{\partial \mathbf{r}} - \left(-\frac{d\pi}{d\mathbf{r}} + \beta\frac{d(\rho K)}{d\rho}\mathbf{u}\right)\rho_l' - \beta\rho K(\mathbf{v}_l' - \mathbf{w}_l') = \mathbf{F}_l^{(i)}$$

$$\rho\frac{\partial \mathbf{w}_l'}{\partial t} = -\rho\frac{\partial \pi_l'}{\partial \mathbf{r}} + \left(-\frac{d\pi}{d\mathbf{r}} + \mathbf{g} + \beta\frac{d(\rho K)}{d\rho}\mathbf{u}\right)\rho_l' + \frac{\partial(\rho\sigma_l')}{\partial \mathbf{r}} + \tag{1.1}$$
$$+\beta\rho K(\mathbf{v}_l' - \mathbf{w}_l') - \mathbf{F}_l^{(p)} + \mathbf{F}_l^{(i)}, \quad K = K(\rho), \quad \beta = \frac{9\varkappa\nu_0}{2a^2}, \quad \varkappa = \frac{d_1}{d_2}$$

The tensor σ_l' describes the stresses due to the action on the pulsation under consideration of perturbations whose scale is smaller than l. The effective viscosity produced by such perturbations can be assumed to be quite small, which allows us to omit the term containing σ_l' in (1.1). This is equivalent physically to the transfer of energy from the pulsations in question to the "quiverings" of the particles and gas within the limits of individual specific volumes.

The normal stress tensor π' is not necessarily spherical in the general case (see the discussion in [2]). The use of a specifically spherical tensor π' in [1] is due to the instrument approximations subsumed by the hypothesis adopted in [1], whereby it is possible to describe explicitly the perturbations of arbitrarily small scale within the context of the unified equations of continuous media.

The quantities Q and F in the right sides of Eqs. (1.1) are random functions; the time scale τ of their essential variation is much smaller than the scale of decay T of the correlation functions. Hence, in analyzing processes occurring in the time $t \gg \tau$ (but $t \ll T$ or $t \gtrsim T$) these quantities can be considered as Markovian random functions of time. This fact also makes possible a stricter justification of the method of [2]. In fact, by averaging Eqs. (1.1) over time intervals $t \gg \tau$, we arrive directly at equations of the same type as in [2], which enable us to describe the regular degeneration of the fluctuation field by solving a certain problem under initial conditions.

As is noted in [1], the quantities τ and T are similar in meaning to the internal and external time scales of turbulence of a monophase fluid which were introduced in [3]. Moreover, the quantity τ can be regarded as the characteristic interaction time τ_i in the statistical system under consideration, so that the assumption that the random quantities Q and F of (1.1) are Markovian is equivalent to the familiar asymptotic case $t \gg \tau_i$ in nonequilibrium statistical mechanics (e.g. see [4]). The relationship between (1.1) and the corresponding averaged equations has the same significance as the relationship between the Liouville equation and the equations which result from it upon approximation of the random phases (e.g. by a controlling equation or the Boltzmann equation).

Neglecting σ_l', we obtain from (1.1) the following equation for π_l' (from now on we omit the subscript l):

$$-(1-\rho)\frac{\partial^2\pi'}{\partial\mathbf{r}\,\partial\mathbf{r}} = \frac{\beta\rho K}{1-\rho}\frac{\partial\rho'}{\partial t} + \left[-\frac{d\pi}{d\mathbf{r}} + \beta K\left(\frac{1}{1-\rho} + \rho\frac{d\ln K}{d\rho}\right)\mathbf{u}\right]\frac{\partial\rho'}{\partial\mathbf{r}} -$$
$$-\beta\rho K\frac{\partial\mathbf{w}'}{\partial\mathbf{r}} + \frac{\partial\mathbf{F}^{(i)}}{\partial\mathbf{r}} - \frac{\beta\rho K}{1-\rho}Q^{(g)} \tag{1.2}$$

Expressing π' as the sum $\pi_0'\mathrm{I} + \pi_1'$, where I is a unit tensor, applying self-evident symmetry considerations, and retaining the isotropic term in the right side of (1.2) in the equation for π_0' and the anisotropic term in the right side of (1.2) in the equation for π_1', we obtain the following expression for $\nabla\pi_1'$:

$$-\frac{\partial\pi_1'}{\partial\mathbf{r}} = \frac{1}{1-\rho}\left[-\frac{d\pi}{d\mathbf{r}} + \beta K\left(\frac{1}{1-\rho} + \rho\frac{d\ln K}{d\rho}\right)\mathbf{u}\right]\rho' - \frac{\beta\rho K}{1-\rho}\mathbf{w}' + \frac{\mathbf{F}^{(i)}}{1-\rho} \tag{1.3}$$

Making use of (1.3) and carrying out some transformations of (1.1), we obtain

$$\frac{\partial^2\rho'}{\partial t^2} + b_1\frac{\partial\rho'}{\partial t} + \mathbf{W}\frac{\partial\rho'}{\partial\mathbf{r}} = \frac{\partial}{\partial\mathbf{r}}\left(\mathbf{F}^{(p)} - \frac{\mathbf{F}^{(i)}}{1-\rho}\right) + \frac{\beta\rho K}{1-\rho}\left(\frac{Q^{(p)}}{\rho} + \frac{Q^{(g)}}{1-\rho}\right) + \frac{\partial Q^{(p)}}{\partial t} \tag{1.4}$$

In accordance with (1. 1) and (1. 2), the equations for π_0', $\mathbf{w}'$ and $\mathbf{v}'$ are of the form

$$-(1-\rho)\frac{\partial^2\pi_0'}{\partial \mathbf{r}\,\partial \mathbf{r}} = \frac{\beta\rho K}{1-\rho}\left(\frac{\partial \rho'}{\partial t} - Q^{(g)}\right), \qquad \rho\left(\frac{\partial}{\partial t} + \frac{\beta K}{1-\rho}\right)\mathbf{w}' + \frac{\partial \pi_0'}{\partial \mathbf{r}} =$$

$$= \mathbf{W}\rho' - \mathbf{F}^{(p)} + \frac{\mathbf{F}^{(i)}}{1-\rho}, \qquad \beta\rho K\mathbf{v}' + (1-\rho)\frac{\partial \pi_0'}{\partial \mathbf{r}} = \frac{\beta\rho K}{1-\rho}\mathbf{u}\rho' \tag{1.5}$$

$$\mathbf{W} = -\frac{1}{1-\rho}\frac{d\pi}{d\mathbf{r}} + \mathbf{g} + \frac{\beta K}{1-\rho}\left(\frac{1}{1-\rho} + \rho\frac{d\ln K}{d\rho}\right)\mathbf{u}, \qquad b_1 = \frac{\beta K}{(1-\rho)^2}$$

In dealing with this problem we are interested only in the quasiturbulent motions occasioned by the interaction of the carrier current and gravitational forces on the one hand, and the fluctuations of the system concentration on the other, and not in the ordinary turbulent motions of the phases. For this reason we need merely investigate the particular "natural" solution of system (1. 4), (1. 5) which is determined by the presence of the random functions Q and $\mathbf{F}$ in the right sides of the equations.

Let us compare the distinct terms in the right side of Eq. (1. 4). The last term is clearly of the same order as the quantity $\tau^{-1}\,Q^{(p)}$, i. e. in the limiting case $\tau \to 0$ it is considerably larger than the first two terms (we assume that the mean-square values of the random functions Q, F_i are of the same order in τ). From this we infer that in our "Markovian" approximation we need only take account of the terms containing the function $Q^{(p)}$, neglecting the terms containing $\mathbf{F}$ and $Q^{(g)}$. It is clear, moreover, that $Q^{(p)}$ describes precisely that statistical noise (white with respect to time) which was introduced in [2] on the basis of intuitive physical considerations.

Let us introduce the notions of random processes in the form of Fourier-Stieltjes stochastic integrals. We then obtain from (1. 4) the following equation for the random measure dZ_ρ of the process ρ' :

$$dZ_\rho = (i\omega^2 + b_1\omega + b_2)^{-1}\, dC, \qquad b_2 = \mathbf{Wk} \tag{1.6}$$

Here the quantity dC represents the random measure of the process which is Markovian with respect to time. We note that (1. 6) coincides with the analogous equation of [2] as the phase density ratio $\varkappa$ approaches zero.

Again neglecting the quantities $Q^{(g)}$, $\mathbf{F}^{(i)}$ and $\mathbf{F}^{(p)}$, we obtain from (1. 5) the following expressions for the random measures of the processes π_0', $\mathbf{v}'$ and $\mathbf{w}'$:

$$dZ_{\pi 0} = i\rho b_1\frac{\omega}{k^2}dZ_\rho, \qquad dZ_v = \frac{1}{1-\rho}\left(\mathbf{u} + \frac{\omega\mathbf{k}}{k^2}\right)dZ_\rho$$

$$dZ_w = \frac{1}{\rho\,(i\omega+\omega_0)}\left(\mathbf{W} + \rho b_1\frac{\omega\mathbf{k}}{k^2}\right)dZ_\rho, \qquad \omega_0 = \frac{\beta K}{1-\rho} \tag{1.7}$$

These expressions are also equivalent to the expressions of [2]. The pulsation of the dispersion medium is secondary in the sense that its appearance is itself due to the necessity of mass conservation in the chaotic particle motions and to the resulting fluctuations of particle concentrations in the system (see the discussion in [2]). It is also quite simple to write out an expression for the quantity $\mathbf{k}dZ_{\pi_1}$, which is a consequence of (1. 3).

We note that the same results are readily obtainable for a particle suspension in a liquid, i. e. when one cannot neglect momentum and viscous energy dissipation in the dispersion medium. The method of [1] combined with the ideas employed above also yields the equations written out in [2] in this case.

Further on we shall carry out several sample computations for one-dimensional gradientless steady flow. The dynamic equations of [1] yield the following relations for such flow:

$$\frac{d\pi}{d\mathbf{r}} = \frac{\mathbf{g}}{g}\frac{d\pi}{dx_3} = \rho\mathbf{g}, \quad \mathbf{u} = -\frac{(1-\rho)\,\mathbf{g}}{\beta K}, \quad \mathbf{W} = -\rho\mathbf{g}\left(\frac{2}{1-\rho} + \frac{d\ln K}{d\rho}\right) \tag{1.8}$$

These relations enable us to describe such flows by specifying a single dynamic quantity.

2. The structure of equilibrium states. According to [2], the spectral density $\Psi_{\rho,\rho}(\omega, \mathbf{k})$ of the random process ρ can be expressed by means of (1.6) in terms of the spectral density $\Phi_{\rho,\rho}(\mathbf{k})$ of this process which describes the simultaneous correlation functions only. The position of an individual particle in the model under consideration is defined to within its specific volume. Allowance for this indeterminacy by means of the Massignon procedure for smoothing out short-wave spectral details enabled us [5] to obtain an expression for $\Phi_{\rho,\rho}(\mathbf{k})$ in a system of statistically independent particles. As we noted in [2], it is possible to use some equivalent method rather than that of Massignon. Specifically, by using the simplest spectrum smoothing procedure based on the familiar ideas of Debye, we arrive in the same way at the relations [2,5]

$$\Psi_{\rho,\rho}(\omega, \mathbf{k}) = \frac{\Phi_{\rho,\rho}(k)}{\omega^4 + (b_1\omega + b_2)^2}\left(\int_{-\infty}^{\infty} \frac{d\omega}{\omega^4 + (b_1\omega + b_2)^2}\right)^{-1}$$
$$\Phi_{\rho,\rho}(k) = \frac{3}{4\pi}\frac{\rho^2}{k_0^3}\left(1 - \frac{\rho}{\rho_*}\right) Y(k_0 - k) \qquad (2.1)$$

Here $Y(x)$ is Heaviside's function, ρ_* is the concentration of the gas suspension in the dense packing state, and the quantity k_0 is simply related to the dimension of the independent statistical unit in the system. In a system of statistically independent particles the role of this unit is played by the particle with its specific volume, so that

$$k_0 = k_\infty = (^3/_4\,\pi\rho)^{1/3} a^{-1} = (^3/_4\pi)^{1/3} b^{-1}, \qquad b = a\rho^{-1/3} \qquad (2.2)$$

The hypothesis of statistical independence is not adequate in the general case (an approximate criterion of appearance of large-scale perturbations and transition to an inhomogeneous flow state is formulated in [2]). The existence of correlations between the behavior of neighboring particles clearly results in this case in an increase of the independent statistical unit in the stream, so that k_0 in (2.1) turns out to be smaller than the quantity k_∞ in (2.2). For this reason we shall be considering k_0 as some function of the dynamic quantities and physical parameters of the phases.

Roughly speaking, the use of Formula (2.1) for $k_0 < k_\infty$ corresponds to the notion of a gas suspension as a system consisting of groups of particles whose behavior within a single group is completely correlated, even though the groups themselves are statistically independent. The quantity b_0, which is related to k_0 as b is to k_∞, in (2.2) then represents the radius of the volume occupied by such a group and therefore determines both the scale of long-range interaction in the system and the scale of the resulting inhomogeneities. We are referring, of course, to the structural inhomogeneities due to the statistical properties of local interactions in the system, and not to the perturbations which can result from a disruption of the hydrodynamic stability of the stream.

We note that to within constant factors of order unity the same results (2.1) and (2.2) can be arrived at by means of a more conventional argument based on equating the number of harmonics in the Fourier transforms of the random processes to the number of degrees of freedom of the particles in the volume element under consideration. From this standpoint the appearance of correlations between particles is equivalent to a reduction of the number of degrees of freedom in the system.

It is also possible in theory to use a phenomenological approach analogous to the well-known Ornstein-Zernike method in the theory of critical fluctuations of dense gases (e.g. see [6]), i.e. to use some rational functional relation for $\Phi_{\rho\rho}(k)$, which contains one or several empirical parameters. For comparison, we shall later make use of a spectral density of this type associated with the Gaussian correlation function $Q_{\rho\rho}(\xi)$ of the process ρ', i.e.

$$Q_{\rho,\rho}(\xi) = \rho^2\left(1 - \frac{\rho}{\rho_*}\right)\exp\frac{-k_0^2\xi^2}{4}, \quad \Phi_{\rho,\rho}(k) = \frac{1}{\pi^{3/2}}\frac{\rho^2}{k_0^3}\left(1 - \frac{\rho}{\rho_*}\right)\exp\frac{-k^2}{k_0^2} \qquad (2.3)$$

The parameter k_0 has the same meaning in (2.3) as in (2.1).

Part of the pulsation energy is dissipated as a result of energy transfer from the perturbations in question to the small-scale "quiverings" of particles within the limits of their specific volumes. This transfer occurs by way of direct and indirect particle collisions[5].

If the flow is inhomogeneous, i.e. if the pulsation scales are small as compared with the specific-volume radius, then the effect of the small-scale "quiverings" can be described by introducing the effective viscosity occasioned by them. Assuming that the motions occurring within the limits of the specific volumes are approximately isotropic and denoting the aforementioned viscosity by ν_m, we obtain the following equation for the energy dissipation due to the small-scale motions:

$$\varepsilon_m = \rho d_2 \nu_m \int\limits_{\omega} \int\limits_{\mathbf{k}} \left(k^2 \Psi_{wi,\, wi} + \frac{1}{3} k_i k_j \Psi_{wi,\, wj} \right) d\omega\, d\mathbf{k} \tag{2.4}$$

$$\Psi_{wi,\, wj}(\omega, \mathbf{k})\, d\omega\, d\mathbf{k} = \langle dZ_{wi}^{*} dZ_{wj} \rangle$$

Relation (2.4) differs from the expression of [5] in that the former retains the terms associated with the "compressibility" of the dispersed phase, i.e. in the fact that the divergence of $\mathbf{w}'$ differs from zero. The notion that the particle pulsations within the specific volumes are isotropic is in a certain sense analogous to the hypothesis of local isotropy in turbulence theory and is confirmed by direct observations (e.g. see [7]). We note, incidentally, that our conclusion concerning the isotropic character of the partial spectral density $\Phi_{\rho,\rho}$ (k) is consistent with the well-known results whereby the three-dimensional correlations of gas density do not depend in the first approximation on the intensity of the generalized thermodynamic forces [6].

Clearly, the quantity ε_m must equal the energy dissipation occurring by way of the small-scale "quiverings" as a result of viscous interaction with the gas. For the latter quantity we use, as in [5], an expression which follows from the theory of Brownian motion, i.e. $\varepsilon_m = 3\rho d_2 \beta^2 K_1^2$, where K_1 (ρ) is a function which corrects for the effect of boundedness of the specific volume on the viscous interaction force; it is analogous to the function K (ρ) in (1.1). Generally speaking, K_1 $(\rho) \neq K$ (ρ). Equating our two expressions for ε_m, we obtain the equation

$$3\beta^2 K_1^2 = I = \int\limits_{\omega} \int\limits_{\mathbf{k}} \left(k^2 \Psi_{wi,\, wi} + \frac{1}{3} k_i k_j \Psi_{wi,\, wj} \right) d\omega\, d\mathbf{k} \tag{2.5}$$

This equation must be used in determining the parameter k_0 in the expressions for $\Phi_{\rho,\rho}(k)$ of (2.1) or (2.3).

The above results enable us to express all of the correlation functions which are of interest as quadratures in ω and $\mathbf{k}$. Actual integration involves difficulties occasioned by the complex form of the expression for $\Psi_{\rho,\rho}$ $(\omega, \mathbf{k})$ in (2.1). We shall consider below just one limiting case in which the integration is simplified considerably. In the general case our results can be regarded as model results of a sort.

The case of greatest interest is the inhomogeneous flow of a gas suspension which arises [2] in the range of parameter values where $|b_2|$ in (2.1) is larger than b_1^2. In the limiting case $b_1^2 \ll |b_2|$ in the main portion of the space of integration over k we find from (2.1) that

$$\Psi_{\rho,\rho}(\omega, \mathbf{k}) \approx \frac{\sqrt{2}}{\pi} \frac{W^{3/2} |k_1|^{3/2}}{\omega^4 + W^2 k_1^2} \Phi_{\rho,\rho}(k), \quad |b_2| \sim W k_0 \gg b_1^2 \tag{2.6}$$

The direction of the axis $x_1 = x$ is the same as that of the vector $\mathbf{W}$.

In the opposite limiting case we have

$$\Psi_{\rho,\rho}(\omega, \mathbf{k}) \approx \frac{2^{-1/2} b_1^{-1} W^2 k_1^2 \Phi_{\rho,\rho}(k)}{\pi(\omega^2 + b_1^2)\left[(\omega + W k_1 / b_1)^2 + {}^1/_2 W^4 k_1^4 b_1^{-3}\right]^2} \tag{2.7}$$

instead of (2.6).

Asymptotic form (2.6) is associated with an approximate solution of Eq. (2.5) of the form

$$k_0 \approx 1.30 \frac{\beta^2 K^{2/3} K_1^{4/3}}{W\left[(1-\rho)(1-\rho/\rho_*)\right]^{2/3}}$$

where we make use of $\Phi_{\rho\rho}(k)$ from (2.1).

In carrying out the integration in (2.5) we took account of only the largest anisotropic term in the expression for the spectral tensor $\Psi_{w i, w j}(\omega, k)$ of the process w' and made use of the relation

$$\int_{-\infty}^{\infty} \frac{d\omega}{(\omega^2+\omega_0^2)(\omega^4+W^2k_1^2)} \approx \frac{1}{W^2k_1^2} \int_{-\infty}^{\infty} \frac{d\omega}{\omega^2+\omega_0^2} = \frac{\pi}{\omega_0 W^2 k_1^2}$$

By using $\Phi_{\rho,\rho}(k)$ from (2.3), we obtain a formula of the same type for k_0, but with the numerical coefficient 0.825 instead of 1.30. Hence, in the general case the indicated formula is valid only to within a factor of order unity. Specifically, for a flow described by relations (1.8) we have the expressions

$$k_0 \approx CG(\rho)u^{-2}g \equiv CG_0(\rho)\,k_*, \qquad C \sim 1, \qquad k_* = u_0^{-2}g, \quad u_0 = g/\beta$$

$$G(\rho) = \frac{(1-\rho)^{4/3}}{\rho(1-\rho/\rho_*)^{2/3}}\left(\frac{2}{1-\rho}+\frac{d\ln K}{d\rho}\right)^{-1}\left(\frac{K_1}{K}\right)^{4/3}, \quad G_0(\rho) = G(\rho)\left(\frac{K}{1-\rho}\right)^2 \qquad (2.8)$$

From this we see that basic assumption (2.6) can be fulfilled either for sufficiently small ρ or for ρ which are quite close to ρ_*. In the general case Formula (2.8) yields the order of the true value of k_0. For example, in the opposite asymptotic case (2.7) we obtain from (2.5) the following expression for k_0 :

$$k_0 \approx CG'(\rho)\,u^{-2}g, \quad G'(\rho) = \frac{1-\rho}{\left[1-0.80\rho+0.28\rho^2(2-\rho)(1-\rho)^{-1}\right]^{1/2}} \times$$
$$\times\left[\rho\left(1-\frac{\rho}{\rho_*}\right)^{1/2}\left(\frac{2}{1-\rho}+\frac{d\ln K}{d\rho}\right)\right]^{-1}\frac{K_1}{K} \qquad (2.9)$$

It is clear that the values of the function $G'(\rho)$ are of the same order as the values of $G(\rho)$ from (2.8) for almost all ρ. We note that $G(\rho)$ and $G'(\rho)$ in (2.8) and (2.9) are weakly dependent on the physical parameters of the phases and on the dynamic variables. This is due to the analogous dependence of the function $K(\rho)$, for which we can take, for example, $K(\rho) \approx (1-\rho)^{-n}$, where the parameter n assumes different values in the various ranges of variation of the dimensionless criteria describing the motion [8].

Relations (2.8) and (2.9) are valid for $k_0 < k_\infty$, when introduction of the viscosity ν_m, and therefore Eq. (2.5), are valid. If the k_0 in (2.8) or (2.9) is larger or equal to the k_∞ of (2.2), then we must take $k_0 \equiv k_\infty$. It is clear that disruption of the homogeneity of the gas suspension stream is facilitated for intermediate values of ρ from the interval $(0, \rho_*)$. The flow of any gas suspension characterized by an arbitrarily small k_* from (2.8) becomes homogeneous as $\rho \to 0$ and $\rho \to \rho_*$. The latter statement is qualitatively consistent with the results of numerous experiments on the quasifluidization of solid particles. In fact, a particle layer is homogeneous near the start of pseudofluidization or with very large fluxes of the suspending gas even if the resulting suspended particle layer of these same particles is essentially inhomogeneous [9].

The condition of homogeneous flow of a gas suspension can be written as $k_0 \gtrsim k_\infty$. Recalling that $ak_* \sim A$, where A is the Archimedes number, we readily obtain this condition from (2.2) and (2.8) in a form similar to that of the approximate homogeneity criterion in [2]. The criterion of the start of an inhomogeneous flow of a given particle system in a given gas (i.e. for a fixed k_*) for an arbitrary concentration ρ can be written as

$$\min_\rho (k_0 - k_\infty) < 0$$

Carrying out some simple transformations and recalling that min $\{\rho^{-1/3} G\} \sim 1$, we obtain this criterion in the form

$$F = u^2 (ag)^{-1} \gtrsim 1 \tag{2.10}$$

where F is the dimensionless Froude flow parameter. Inequality (2.10) coincides in all particulars with the empirical criterion of disruption of homogeneous quasifluidization established in [10] on the basis of a large body of experimental data.

Expressions (2.2) and (2.8) yield the following relation for the size b_0 of the flow inhomogeneities:

$$b_0 \approx \frac{1}{C}\left(\frac{3\pi}{2}\right)^{1/3} \frac{F}{G(\rho)} \equiv \frac{1}{C}\left(\frac{3\pi}{2}\right)^{1/3} \frac{F_0}{G_0(\rho)}, \qquad F_0 = \frac{u_0^2}{ag} = \frac{1}{ak_*} \tag{2.11}$$

This expression has the same structure as the formula for the diameter of a gas bubble in a suspended layer obtained in [9] by analyzing the stability of such a bubble. Its meaning is altogether different, however. In fact, (2.11) describes the size of the inhomogeneities whose appearance is due to the peculiarities of local interactions in the system; the formula of [9], on the other hand, describes the size of hydrodynamic perturbations of a specific type. The coefficient in front of the Froude number F_0 in (2.11) is proportional to $G_0^{-1}(\rho)$ and is usually much smaller than the coefficient in [9], which varies from 200 to 11,000. This shows that large stable hydrodynamic perturbations can occur even in relatively homogeneous streams. This lends added importance to the study of the stability of gas suspensions considered in the continuous-medium approximation. The progressive growth of such perturbations at first introduced externally into a homogeneous suspended layer was observed, for example, in [11].

Asymptotic form (2.6) is associated with the following expressions for the mean-square pulsations of the velocities of the phases (the corresponding spectral velocities can be determined from (1.7)):

$$\langle v_{2'}'^2\rangle \equiv \langle v_{3'}'^2\rangle \approx 0.187\left(\frac{\rho}{1-\rho}\right)^2\left(1-\frac{\rho}{\rho_*}\right)\frac{W}{k_0}\delta^2, \quad \langle v_{1'}'^2\rangle \approx 2\langle v_{2'}'^2\rangle +$$
$$+\left(\frac{\rho}{1-\rho}\right)^2\left(1-\frac{\rho}{\rho_*}\right)u^2, \quad \langle w_2'^2\rangle \equiv \langle w_3'^2\rangle \approx \rho^2\left(1-\frac{\rho}{\rho_*}\right)\frac{b_1^2}{k_0^2} \tag{2.12}$$
$$\langle w_1'^2\rangle \approx \langle w_2'^2\rangle + 3.40\left(1-\frac{\rho}{\rho_*}\right)\frac{W^{2/3}}{\omega_0 k_0^{1/3}}, \quad \delta = \cos(x_1 x_1')$$

Here the components $v'_i{}'$ are taken along the axes x_i' such that $x' = x'_1$ is directed along the vector $\mathbf{u}$. It is clear that the axes x_i and x_i' are not necessarily coincident.

Specifically, we obtain the following expressions from (2.12) for the pulsations in stream (1.8) with k_0 from (2.8):

$$\langle v_2'^2\rangle \equiv \langle v_3'^2\rangle \approx V_2(\rho)\,u^2, \qquad \langle v_1'^2\rangle \approx V_1(\rho)\,u^2$$
$$\langle w_2'^2\rangle \equiv \langle w_3'^2\rangle \approx W_2(\rho)\,u^2, \qquad \langle w_1'^2\rangle \approx W_1(\rho)\,u^2$$
$$V_1(\rho) = 2V_2(\rho) + \left(\frac{\rho}{1-\rho}\right)^2\left(1-\frac{\rho}{\rho_*}\right)$$
$$V_2(\rho) = \frac{0.144\rho^4}{(1-\rho)^{10/3}}\left(1-\frac{\rho}{\rho_*}\right)^{2/3}\left(\frac{2}{1-\rho}+\frac{d\ln K}{d\rho}\right)^2\left(\frac{K}{K_1}\right)^{4/3} \tag{2.13}$$
$$W_1(\rho) = W_2(\rho) + \frac{3.0\rho^2}{(1-\rho)^{2/3}}\left(1-\frac{\rho}{\rho_*}\right)^{4/3}\left(\frac{2}{1-\rho}+\frac{d\ln K}{d\rho}\right)^2\left(\frac{K}{K_1}\right)^{2/3}$$
$$W_2(\rho) = \frac{0.592\rho^4}{(1-\rho)^{14/3}}\left(1-\frac{\rho}{\rho_*}\right)^{1/3}\left(\frac{2}{1-\rho}+\frac{d\ln K}{d\rho}\right)^2\left(\frac{K}{K_1}\right)^{8/3}$$

The ratios

$$N_v = \frac{\langle v_2'^2 \rangle}{\langle v_1'^2 \rangle}, \qquad N_w = \frac{\langle w_2'^2 \rangle}{\langle w_1'^2 \rangle}$$

which can be readily determined from (2.12) and (2.13), are usually important flow characteristics. The quantity N_w is considerably smaller than unity; the quantity N_v turns out to be quite close to 0.5 for certain ρ. Comparison of the theoretical values of $\langle w_1'^2 \rangle$ and N_w with the experimental data of [12,13] indicated good agreement despite the clearly approximate character of relations (2.13). The theoretical values are also qualitatively consistent with other experimental findings on the pulsations of the phases in quasifluidized systems.

As ρ approaches zero or ρ_*, the quantity k_0 from (2.8) is replaced by k_∞ from (2.2), no matter how small the quantity k_* in (2.8). The corresponding expressions for the mean squares of the pulsations are also readily obtainable from (2.12). It can be shown that condition (2.6) is fulfilled in the range $\rho \sim \rho_*$ and condition (2.7) in the range $\rho \sim 0$. As $\rho \to \rho_*$ we have the estimates

$$\langle v_i'^2 \rangle \sim \langle w_i'^2 \rangle \sim 1 - \rho / \rho_* \tag{2.14}$$

The same method can be used to find expressions for $\langle v_i'^2 \rangle$ and $\langle w_i'^2 \rangle$ corresponding to asymptotic form (2.7), and also to write out integral expressions for various correlation functions and to carry out the integrations in them for various specific cases.

The tensor of the dispersed-phase pressure introduced in [1] is diagonal in our coordinate system. Neglecting the volume occupied by the particles themselves, we obtain the equations

$$P_{11} = \rho d_2 \langle w_1'^2 \rangle, \qquad P_{22} = P_{33} = \rho d_2 \langle w_2'^2 \rangle, \qquad P_{ij} = 0, \qquad i \neq j \tag{2.15}$$

These relations correspond to pure quasiturbulent motion [1]. In order to allow for the increasing role of direct interactions between particles with increasing system concentration, we introduced in [2] the notion of a quasigaseous state; the effective normal stresses and the transfer coefficients in the dispersed phase in this state were estimated on the basis of Enskog's results for a dense rigid-sphere gas [14].

In fact, our notions of the direct collisions of gases similar in type to the collisions of gas molecules are in large measure arbitrary even in the case of concentrated large-particle suspensions; this was noted, for example in [6,7]. There are, generally speaking, no grounds to suppose that Enskog's relations describe transfer processes in a disperse system with sufficient accuracy. It is equally doubtful whether Boltzmann-type equations are at all applicable to such systems [3,5]. In the discussion to follow we shall make allowance for the intrinsic volume of the particles by means of an elementary "geometric" method already used with gases [14].

The normal stresses P_{ii} in the dispersed phase are equal to the velocities of transfer of the momentum density of this phase in the i-th directions. The velocity of momentum transfer in the free volume of the particles is clearly equal to the particle pulsation velocity $\langle w_i'^2 \rangle^{1/2} \sim w_i^*$. The velocity c of momentum propagation in the particle material is approximately equal to the velocity of sound, i.e. it is much larger than w_i^*. The time $t_L^{(i)}$ required for the momentum to travel the distance L in the i-th direction is given by

$$t_L^{(i)} \approx \frac{\sigma^{1/3} - \sigma_*^{1/3}}{\sigma^{1/3}} t_1 + \frac{\sigma_*^{1/3}}{\sigma^{1/3}} t_2, \qquad t_1 \approx \frac{L}{w_i^*}, \qquad t_2 \approx \frac{L}{c} \tag{2.16}$$

Here σ_* is the specific volume of a particle in the dense packing state, so that $\sigma - \sigma_*$ is the free volume of the gas suspension per particle. Expression (2.16) readily yields an expression for the mean velocity of momentum propagation. The usual procedure [14] can then be used to find an expression for P_{ii} to replace (2.15),

$$P_{11} \approx \rho d_2 \langle w_1'^2 \rangle \varphi_1, \qquad P_{22} \equiv P_{33} \approx \rho d_2 \langle w_2'^2 \rangle \varphi_2$$

$$\varphi_i = \frac{c}{(1-\gamma)c + \gamma w_i^*} \approx \frac{1}{1-\gamma}, \qquad \gamma = \left(\frac{\sigma_*}{\sigma}\right)^{1/3} = \left(\frac{\rho}{\rho_*}\right)^{1/3} \tag{2.17}$$

The effective particle diffusion coefficients in a quasiturbulent state can be expressed approximately as products of the corresponding velocities and spatial scales of the pulsations. These scales can be estimated in various ways (one of them is described in [2]). For simplicity, we shall limit ourselves here to the formal representations

$$D_{11} = \langle w_1'^2 \rangle^{1/2} L_1 \sim w_1^* b_0, \quad D_{22} \equiv D_{33} \sim w_2^* b_0 \tag{2.18}$$

The coefficients of momentum and pulsation energy transfer, μ_{ii} and λ_{ii}, respectively, can be expressed in terms of D_{ii},

$$\mu_{ii} \equiv \lambda_{ii} \approx \rho d_2 D_{ii} \varphi_i \tag{2.19}$$

Here we have made allowance for the practically instantaneous momentum and energy transfer in the particle material.

For $\rho \to \rho_*$ we obtain from (2.14) and (2.17) the estimates

$$P_{ii} \sim (1 - \gamma^3)\left[(1-\gamma) + \frac{w_i^*}{c}\gamma\right]^{-1}, \qquad \gamma^3 = \frac{\rho}{\rho_*}$$

If $1 - \gamma \gg w_i^* / c$, then $P_{ii} \to$ const as $\rho \to \rho_* (\gamma \to 1)$. However, for ρ so close to ρ_* that $1 - \gamma \ll w_i^*/c$ we have $P_{ii} \to 0$. The range of ρ values adjacent to ρ_* for which the latter inequality is fulfilled will not be taken into account from now on. We then obtain the following estimates for D_{ii} and μ_{ii} from (2.14) and (2.18), (2.19) as $\rho \to \rho_*$:

$$D_{ii} \sim (1-\gamma^3)^{1/2} \to 0, \qquad \mu_{ii} \sim (1-\gamma^3)^{-1/2} \to \infty, \quad \gamma \to 1$$

Here we have taken account of the fact that near ρ_* the quantity $b_0 = b$ from (2.2). It is easy to show that the quantities P_{ii} and D_{ii} considered as functions of ρ usually have maxima, and that μ_{ii} and λ_{ii} have both maxima and minima, as was already noted in [2].

3. The energy equations. The level of development of phase pulsations is determined by the balance existing between the energy expended by the carrier current and gravitational field on the acceleration of individual particles and aggregates (packets) consisting of more than one particle, and the dissipation of the pulsation energy through viscous particle-gas interactions (*).

Neglecting the gas pulsation energy, we obtain from the second and third equations of (1.5) the following balance equation for the quantities $\langle w_i'^2 \rangle$ (without summation over i):

$$\frac{1}{2}\rho\frac{\partial \langle w_i'^2\rangle}{\partial t} = \rho R_i \langle \rho' w_i' \rangle + \frac{\beta\rho K}{1-\rho}\langle v_i' w_i' \rangle - \frac{\beta\rho K}{1-\rho}\langle w_i'^2 \rangle \tag{3.1}$$

$$\rho \mathbf{R} = \mathbf{W} - \frac{\beta\rho K}{1-\rho}\mathbf{u} = -\frac{1}{1-\rho}\frac{d\pi}{dr} + \mathbf{g} + \frac{\beta K}{1-\rho}\left(1 + \rho\frac{d\ln K}{d\rho}\right)\mathbf{u}, \qquad \mathbf{w} = 0$$

The first term in the right side of (2.4) describes the work performed by the averaged gas flux and gravitational forces in the fluctuations of the gas suspension concentration; the second term represents the work expended by the gas pulsations in the random particle displacements; the third term represents the dissipation of particle pulsation energy by way of the viscous forces.

Equations (3.1) do not contain terms associated with the irreversible pulsation energy transfer processes (cf. [15]), i.e. terms representing increases in the pulsation energy

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*) Having neglected the tensor σ_i' in Eqs. (1.1), we neglect the small quantity ε_m discussed in detail in Sect. 2.

due to the dissipation of the kinetic energy of the averaged motion and the divergence of the pulsation energy flux due to the transfer of the latter by the pulsations themselves. From the familiar principles of irreversible process thermodynamics we obtain the following expression for the latter flux:

$$q_i = -\frac{1}{2}\left(\lambda \frac{\partial}{\partial \mathbf{r}}\right)\theta_i + \left(\mathbf{m}^{(i,j)} \frac{\partial}{\partial \mathbf{r}}\right)\theta_j, \qquad \theta_i = \langle w_i'^2 \rangle, \quad i \neq j \qquad (3.2)$$

Here θ_i is the doubled kinetic energy of pulsation of a unit mass of the dispersed phase in the i-th direction (the "effective temperature in the i-th direction"); λ is the symmetric tensor of transfer coefficients which coincides with the tensor λ° of (2.19) in the steady ("equilibrium") state (all quantities referring to the equilibrium state will henceforth be accompanied by the degree symbol); $\mathbf{m}^{(i,j)}$ are the cross coefficient tensors symmetric in i, j. For simplicity, we shall ignore all cross effects from now on (by setting $\mathbf{m}^{(i,j)} = 0$).

The increase in pulsation energy due to the dissipation of the energy of averaged motion depends on the derivatives of the dynamic quantities with respect to the coordinates. The corresponding term must therefore be omitted in our gradientless approximation (see the discussion in [16] for the case of a single-phase fluid).

Moreover, a system in a nonequilibrium state can be characterized by exchanges in pulsation energy among the motions in various directions. The characteristic time of such exchanges is clearly $\tau \approx \tau_p$. In our asymptotic case $t \gg \tau$ this exchange must be neglected, since we are considering states which are "relaxed" in the pertinent sense.

From (1.4) we can readily obtain the corresponding equation for $\langle \rho'^2 \rangle$ from which we see that the condition whereby $\langle \rho'^2 \rangle$ is bounded throughout the entire space implies that $\langle \rho'^2 \rangle = \langle \rho'^2 \rangle^\circ$ = const. Hence, recalling what we said in Sect. 1 concerning the secondary character of the gas pulsations, we can write the estimates

$$\rho R_i \langle \rho' w_i' \rangle + \rho \omega_0 \langle v_i' w_i' \rangle \sim \theta_i^{1/2} \qquad (3.3)$$

The proportionality coefficient can be expressed in self-evident fashion in terms of the equilibrium temperature θ_i° on the basis of the condition whereby θ_i becomes θ_i° in the equilibrium state. This way we obtain from (3.1) - (3.3) the following equations for the temperatures θ_i:

$$\left(\frac{\partial}{\partial t} + \mathbf{w}\frac{\partial}{\partial \mathbf{r}}\right)\theta_i = \frac{\partial}{\partial \mathbf{r}}\left(\lambda \frac{\partial}{\partial \mathbf{r}_i}\right)\theta_i + 2\omega_0 \sqrt{\theta_i}\left(\sqrt{\theta_i^\circ} - \sqrt{\theta_i}\right) \qquad (3.4)$$

In order to determine these equations (as well as the dynamic equations of [1]) completely, we must establish the dependence of the tensors λ and μ on θ_i. To this end we note that $I \sim m_i \theta_i$ in (2.5); here m_i are certain functions of k_0, of the dynamic quantities, and of the physical parameters of the system phases. The expression for k_0 in terms of the temperatures θ_i in a nonequilibrium state is therefore of the same form as the expression for k_0° in terms of θ_i° in the equilibrium state. Expressions (2.18) and (2.19) further imply that $\lambda_{ii} \sim \theta_i^{1/2} k_0^{-1}(\theta_i)$ (since $w_*^i \sim \theta_i^{1/2}$) and an analogous expression for μ_{ii}. We can therefore write

$$\lambda_{ii} = \lambda_{ii}^\circ \left(\frac{\theta_i}{\theta_i^\circ}\right)^{1/2} \frac{k_0^\circ(\theta_i^\circ)}{k_0^\circ(\theta_i)}, \qquad \mu_{ii} = \mu_{ii}^\circ \left(\frac{\theta_i}{\theta_i^\circ}\right)^{1/2} \frac{k_0^\circ(\theta_i^\circ)}{k_0^\circ(\theta_i)} \qquad (3.5)$$

which defines the required tensors. The dependence of the normal stresses on the temperature is trivial: we have $P_{ii} = P_{ii}^\circ\,(\theta_i/\theta_i^\circ)$.

For example, for asymptotic case (2.6) we obtain successively

$$k_0 \approx k_0^\circ \left(\frac{\theta_1^\circ}{\theta_1}\right)^{1/4}, \quad \lambda_{11} \approx \lambda_{11}^\circ \left(\frac{\theta_1}{\theta_1^\circ}\right)^{3/4}, \lambda_{22} \equiv \lambda_{33} \approx \lambda_{22}^\circ \left(\frac{\theta_1}{\theta_1^\circ}\right)^{1/4} \left(\frac{\theta_2}{\theta_2^\circ}\right)^{1/2} \qquad (3.6)$$

and similar expressions for μ_{ll}.

We emphasize that the use of Eq.(2.5) to describe nonequilibrium states is equivalent to assuming that equilibrium for the small-scale "quiverings" is established much sooner than for the large-scale pulsations. The validity of this assumption fór steady flows of gas suspensions is self-evident.

The equations for the pulsation energy (3.4) and relations of the type (3.5), (3.6) which, in a sense, play the role of "equations of state", close the system of dynamic equations of [1]. These Equations (3.4) have the same meaning as the heat conduction equation in single-phase fluid hydrodynamics.

After simple transformations, the dynamic equations of [1] yield the equation for the energy of the averaged motion of a gas suspension in the form

$$\left(\frac{\partial}{\partial t} + \mathbf{w}\frac{\partial}{\partial \mathbf{r}}\right)\frac{\rho w^2}{2} = -\frac{w^2}{2}\frac{\partial(\rho\mathbf{w})}{\partial \mathbf{r}} + \mathbf{w}\left[\mathbf{g} + \omega_0\rho(\mathbf{v}-\mathbf{w}) - \frac{\partial(\rho\Pi - \rho\sigma^\circ)}{\partial \mathbf{r}}\right] \quad (3.7)$$

$$\Pi = (\rho d_2)^{-1}\mathbf{P}, \quad \sigma^\circ = (\rho d_2)^{-1}\tau^\circ$$

(the notation here is that of [1]).

Adding (3.7) to the sum of Equations (3.4), we obtain an equation which closes the ordinary heat transfer equation [15]. Equation (3.7) can also be written in other forms.

Equations (3.4) are especially interesting because of the fact that, together with the dynamic equations, they enable us to describe the interaction of a stream with solid walls and the degeneration of pulsations ("cooling" of a gas suspension) in the neighborhood of a wall. This is extremely important both for the formulation of correct boundary conditions at the walls, and especially for computing the coefficients of heat and mass transfer to the wall. The same equations enable us to describe, for example, the rise of chaotic pulsations along the height of the suspended layer, the effect of a gas distribution grating on the structure of the layer, etc. We note that in accordance with what we have said above, the presence of walls affects not only the intensity, but also the scale of the pulsations. This fact explains the growth of inhomogeneities with increasing distance from the distribution grating noted in many experiments.

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SOME GENERAL PROPERTIES OF THE EQUATIONS OF VISCOELASTIC INCOMPRESSIBLE FLUID DYNAMICS

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We consider a system of equations describing the flows of an incompressible viscoelastic medium with a rheological equation of state containing derivatives of the stress tensor with respect to time. The initial system of equations for two-constant models of the medium is a quasilinear first-order system. Correct formulation of the problem under initial conditions requires the imposition of certain restrictions on the system matrix [1]. These restrictions, which are necessary to ensure the evolutionary character of the system, are imposed on the stress tensor in our case. We shall concentrate on one-dimensional motions for which the requirement of evolutionary character renders the system hyperbolic. It is then possible to indicate sufficient conditions which ensure the uniqueness of the continuous solution of the one-dimensional steadystate boundary value problem.

Hyperbolic systems of equations of viscoelastic fluid dynamics have discontinuous solutions for certain models (e.g. that of Oldroyd [2]). Discontinuous flows of materials with memory in which the stresses are functionals of their "strain history" are discussed in [3]. We shall consider the discontinuities in Oldroyd's model when the differential relationship between the stress tensors and straining rates is given. A necessary condition for the existence of discontinuities is formulated. The problem of evolution of a velocity jump in one-dimensional motion is considered.

1. The conditions of evolutionary character. Let a viscoelastic incompressible fluid move in a plane channel $0 < z < \delta$ or in a half-space $z > 0$. We assume that all the parameters of motion except the pressure are functions of the single space coordinate z and of the time t.

The equations of motion are in this case of the form

$$\frac{\partial v_x}{\partial t} + v_z \frac{\partial v_x}{\partial z} - \frac{1}{\rho}\frac{\partial T_{xz}}{\partial z} - \frac{1}{\rho} P_x - F_x = 0$$
$$\frac{\partial v_y}{\partial t} + v_z \frac{\partial v_y}{\partial z} - \frac{1}{\rho}\frac{\partial T_{yz}}{\partial z} - \frac{1}{\rho} P_y - F_y = 0 \qquad (1.1)$$
$$\frac{\partial v_z}{\partial t} - \frac{1}{\rho}\frac{\partial T_{zz}}{\partial z} + \frac{1}{\rho}\frac{\partial p}{\partial z} - F_z = 0$$

Here $P_x(t) = -\partial p / \partial x$ and $P_y(t) = -\partial p/\partial y$ can be regarded as given functions. The last equation of (1.1) makes use of the incompressibility condition